The Scale of 11 and the RJS System

From Generative Structure to Musical Application

by

Delores DeVore

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SoundWorks11| Scale of 11

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Introduction

The material presented in this document represents the result of an extended exploration into frequency, number, and harmonic structure, centered on what is now called the *Scale of 11*. This system did not emerge from modifying existing musical frameworks, but from working directly with frequency relationships based on sequential multiples of 11, beginning with a fixed reference of Middle C at 264 Hz.

During this process, it became clear that commonly referenced frequency systems—often grouped under the term "Solfeggio"—contain inconsistencies in both naming and structure. These inconsistencies led to a deeper investigation into the original numerical patterns associated with Dr. Joseph Puleo's work, and ultimately to a re-examination of the underlying relationships between number, tone, and interval.

What emerged is a two-part framework:

- The **Scale of 11**, which functions as a generative system, producing a continuous set of frequencies through the addition of 11 and its multiples across octaves.
- The **RJS system**, a derived 15-degree structure that reveals a repeating interval pattern within the Scale of 11, providing a practical and portable musical framework.

The charts that follow present these findings in a structured format. The first sections establish the foundational relationships ("The Legion") and display the Scale of 11 across seven octaves. Subsequent sections introduce the RJS system, including its degree relationships, interval structure, and corresponding notation.

This document is intended to clarify the internal logic of the system and provide a reference point for further exploration, application, and development. While the framework is mathematically derived, its purpose is ultimately experiential—bridging number and sound through a consistent harmonic structure.

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The Legion (Foundational Framework)

The Middle C octave serves as the reference point, as it is the first octave in which a complete chromatic structure is fully realized. This octave establishes the standard framework used throughout the system.

Tone Number (Tn) identifies the first appearance of each Tone within the Scale of 11.
Tone numbers are assigned sequentially based on first occurrence, beginning with **F (11Hz) as T1**. Once assigned, a tone retains its Tone Number (Tn) across all octaves.

The musical notation system is anchored to the Middle C octave:

- **C = 264 Hz → C0**
- **D = 297 Hz → D0**
- **C# = 286 Hz → C#0**
- **etc**.

When frequencies that correspond closely to a Middle C reference note appear, they are indexed numerically in order of appearance.

- **C1, C2, C3…**

These indices do **not** indicate octave position, but rather the **order of appearance within the Scale of 11**.

This means that a lower frequency may carry a higher index if its corresponding reference tone appears earlier in the sequence (see D and B examples). The notation reflects **discovery order**, not pitch hierarchy.

Subscript indicates the octave. $C0_4$ = C (264) in octave 4

Key Principles

- Each tone (Tn) is fixed and persists across all octaves.
- Notation (C, D, E, etc.) is derived from the Middle C reference octave.
- Numerical suffixes (0, 1, 2…) track **sequence of emergence**, not octave.
- The system separates:
    - **Tone Identity (Tn)**
    - **Musical Note (C, D, etc.)**
    - **Frequency (Hz)**
    - **Order of Appearance**

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Scale of 11 (S11) First 7 Octaves

Old notation	Musical Notation	Octave -1	Octave 0	Octave 1	Octave 2	Octave 3	Octave 4	Octave 5
						ET Scale Interval = 11 12 unique pitches	Quarter Tone Scale Interval = 11 24 unique pitches	48 unique pitches
C	C0			33	66	132	264	528
								539
C+	C1						275	550
								561
C♯	C♯0					143	286	572
C♯+/ D-	C♯1							583
D	D0						297	594
	D2							605
D+	D1				77	154	308	616
	D4							627
D♯	D♯0						319	638
D♯+/E-	D♯1							649
E	E0					165	330	660
	E2							671
E+	E1						341	682
	E4							693
F	F0	11	22	44	88	176	352	704
	F3							715
F+	F2						363	726
	F4							737
F♯	F♯0					187	374	748
	F♯2							759
F♯+/G-	F♯1						385	770
	F♯3							781
G	G0				99	198	396	792
	G2							803
G+	G1						407	814
	G3							825
G♯	G♯0					209	418	836
	G#2							847
G♯+/A-	G♯1						429	858
	G#3							869
A	A0			55	110	220	440	880
	A1							891

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Scale of 11 First 7 Octaves								
Old notation	Musical Notation	Octave -1	Octave 0	Octave 1	Octave 2	Octave 3	Octave 4	Octave 5
A+	A1						451	902
	A2							913
A♯	A♯0					231	462	924
	A#2							935
A♯+/B-	A♯1						473	946
	A♯3							957
B	B1				121	242	484	968
	B4							979
B+	B0						495	990
	B5							1001
B++	B2					253	506	1012
	B6							1023
B+++	B3						517	1034
	B7							1045
C (Octave)				66	132	264	528	1056
Totals		1, 1 New	1, 0 New	3, 2 New	6, 3 New	12, 6 New	24, 12 New	48, 24 New

Notes: New values will always be odd and will be given a Tone Number. When they show up in octaves above, they will be even. They will not appear in octaves below the current octave where they first show up.

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Scale of 11 – Frequency Mapping

Frequency	Musical Notation	Tone ID	First Octave	Current Octave	Notes
11	$F0_{-1}$	T1	-1	-1	
22	$F0_0$	T1	0	0	
33	$C0_1$	T2	1	1	
44	$F0_1$	T1	-1	1	
55	$A0_1$	T3	1	1	
66	$C0_2$	T2	2	2	
77	$D1_2$	T4	2	2	Although this is the first occurrence of a D-related frequency, it does not correspond to the standard D (297 Hz). Therefore, it is labeled D1, reserving D0 for its appearance in Octave 4.
88	$F0_2$	T1	-1	2	
99	$G0_2$	T5	2	2	
110	$A0_2$	T3	1	2	
121	$B1_2$	T6	2	2	
132	$C0_3$	T2	1	3	
143	$C\sharp0_3$	T7	3	3	
154	$D\sharp0_3$	T4	2	3	
165	$E0_3$	T8	3	3	
176	$F0_3$	T1	-1	3	
187	$F\sharp0_3$	T9	3	3	
198	$G0_3$	T5	2	3	
209	$G\sharp0_3$	T10	3	3	
220	$A0_3$	T3	1	3	
231	$A\sharp0_3$	T11	3	3	
242	$B1_3$	T6	2	3	
253	$B2_3$	T12	3	3	
264	$C0_4$	T2	1	4	
275	$C1_4$	T13	4	4	
286	$C\sharp0_4$	T7	3	4	
297	$D0_4$	T14	4	4	
308	$D1_4$	T4	2	4	
319	$D\sharp0_4$	T15	4	4	
330	$E0_4$	T8	3	4	
341	$E1_4$	T16	4	4	
352	$F0_4$	T1	-1	4	
363	$F1_4$	T17	4	4	
374	$F\sharp0_4$	T9	3	4	
385	$F\sharp1_4$	T18	4	4	

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Scale of 11 – Frequency Mapping

Frequency	Musical Notation	Tone ID	First Octave	Current Octave	Notes
396	$G0_4$	T5	2	4	
407	$G1_4$	T19	4	4	
418	$G\sharp0_4$	T10	3	4	
429	$G\sharp1_4$	T20	4	4	
440	$A0_4$	T3	-1	4	
451	$A1_4$	T21	4	4	
462	$A\sharp0_4$	T11	3	4	
473	$A\sharp1_4$	T22	4	4	
484	$B1_4$	T6	2	4	
495	$B0_4$	T23	4	4	B at 495 Hz corresponds to the standard B in the Just scale and appears in Octave 4, where it is assigned the notation B0. The earlier occurrence B1 appears in a lower octave, while B2 and B3 are indexed according to order of appearance in Octave 4.
506	$B2_4$	T12	3	4	
517	$B3_4$	T24	4	4	
528	$C0_4$	T2	1	4	

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Scales Within Octave 4

Equal Temperament (ET)

When working with the Equal Temperament Scale (ET) the Notes do not match the 4th Octave notes.

Equal Temperament Scale (ET)				
Note in Octave 4	ET Note	Freq	Factors	Ratio
$C0_4$	C	264	11x24	24/24 = 1/1
$C\sharp0_4$	C♯	286	11x26	26/24 = 13/12
$D1_4$	D	308	11x28	28/24 = 7/6
$E0_4$	D♯	330	11x30	30/24 = 5/4
$F0_4$	E	352	11x32	32/24 = 4/3
$F\sharp0_4$	F	374	11x34	34/24 = 17/12
$G0_4$	F♯	396	11x36	36/24 = 3/2
$G\sharp0_4$	G	418	11x38	38/24 = 19/12
$A0_4$	G♯	440	11x40	40/24 = 5/3
$A\sharp0_4$	A	462	11x42	42/24 = 7/4
$B1_4$	A♯	484	11x44	44/24 = 11/6
$B2_4$	B	506	11x46	46/24 = 13/12
$C0_5$	C	528	11x48	48/24 = 2/1

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Quarter-Tone Scale

Quarter Tone Scale			
Note	**Freq**	**Factors**	**Ratio**
C0_4	264	11x24	24/24 = 1/1
C1_4	275	11x25	25/24
C♯0_4	286	11x26	26/24 = 13/12
D0_4	297	11x27	27/24 = 9/8
D1_4	308	11x28	28/24 = 7/6
D2_4	319	11x29	29/24
E0_4	330	11x30	30/24 = 5/4
E1_4	341	11x31	31/24
F0_4	352	11x32	32/24 = 4/3
F1_4	363	11x33	33/24
F♯0_4	374	11x34	34/24 = 17/12
F♯1_4	385	11x35	35/24
G0	396	11x36	36/24 = 3/2
G1_4	407	11x37	37/24
G♯0_4	418	11x38	38/24 = 19/12
G♯1_4	429	11x39	39/24 = 13/8
A0_4	440	11x40	40/24 = 5/3
A1_4	451	11x41	41/24
A♯0_4	462	11x42	42/24 = 7/4
B2_4	473	11x43	43/24
B1_4	484	11x44	44/24 = 11/6
B0_4	495	11x45	45/24 = 15/8
B3_4	506	11x46	46/24 = 23/12
B4_4	517	11x47	47/24
C0_5	528	11x48	48/24 = 2/1

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Just Intonation (JI)

Just Intonation Scale (JI)			
Note	Freq	Factors	Ratios
$C0_4$	264	11x24	24/24 = 1/1
$C\sharp_{04}$	286	11x26	26/24 = 13/12
$D0_4$	297	11x27	27/24
$D1\sharp_4$	319	11x29	29/24
$E0_4$	330	11x30	30/24 = 5/4
$F0_4$	352	11x32	32/24 = 4/3
$F\sharp0_4$	374	11x34	34/24 = 17/12
$G0_4$	396	11x36	36/24 = 3/2
$G\sharp0_4$	418	11x38	38/24 = 19/12
$A0_4$	440	11x40	40/24 = 5/3
$A\sharp0_4$	462	11x42	42/24 = 7/4
$B0_4$	495	11x45	45/24= 15/8
$C0_5$	528	11x48	48/24 = 2/1

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RJS System – 15 Note Octave Scale

Key Interpretation Note

The phrases "Song of Degrees," "Song of Degrees of David," and "Song of Degrees for Solomon" are preserved exactly as they appear in the source.

Within this system, these phrases correspond to interval structure:

- **"… of David" → 11 (Short interval)**
- **"… for Solomon" → 22 (Long interval)**
- **"Song of Degrees" → 22 (Long interval)**

This distinction is essential to understanding that the system is based on interval relationships, not note groupings (e.g., black vs. white keys).

These intervals correspond to:

- **11 → Short (S)**
- **22 → Long (L)**

This creates a structural pattern analogous to whole and half steps in traditional scales but based entirely on multiples of 11.

These phonetic tones are placeholders for interval positions, not fixed pitch names.

Resolved Just Scale (RJS) (15 Degrees of the Scale of 11)			
Source Key (Psalms 120–134) Third Clue given to Dr Puleo	Interval	Interval Type (L/S)	Sound
Song of Degrees	22	L	BOH
Song of Degrees	22	L	DOH
Song of Degrees of David	11	S	FAH
Song of Degrees	22	L	GOH
Song of Degrees of David	11	S	HAH
Song of Degrees	22	L	JOH
Song of Degrees	22	L	KOH
Song of Degrees for Solomon	22	L	LOH
Song of Degrees	22	L	MOH
Song of Degrees	22	L	NOH
Song of Degrees	22	L	POH
Song of Degrees of David	11	S	RAH
Song of Degrees	22	L	SOH
Song of Degrees of David	11	S	TAH
Song of Degrees	22	L	VOH
These phrases are preserved exactly as given in the original source and form the key used to derive the RJS system.			

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Resolved Just Scale (RJS)

Traditional Just Intonation contains intervallic gaps (e.g., 33 Hz steps) that are not internally subdivided. The Resolved Just Scale (RJS) reveals that these intervals can be systematically divided into 11 and 22 Hz increments, creating additional tonal steps and a more continuous harmonic structure.

Resolved Just Scale (RJS)

Note	Freq	Factors	Ratios
$C0_4$	264	11X24	24/24 = 1/1
$C\sharp0_4$	286	11X26	26/24 = 13/12
$D0_4$	297	11X27	27/24 = 9/8
$D\sharp0_4$	319	11X29	29/24
$E0_4$	330	11X30	30/24 = 5/4
$F0_4$	352	11X32	32/24 = 4/3
$F\sharp0_4$	374	11X34	34/24 = 17/12
$G0_4$	396	11X36	36/24 = 3/2
$G\#0_4$	418	11X38	38/24 = 19/12
$A0_4$	440	11X40	40/24 = 5/3
$A\sharp0_4$	462	11X42	42/24 = 7/4
$B2_4$	473	11X43	43/24
$B0_4$	495	11X45	45/24 = 15/8
$B3_4$	506	11X46	46/24 = 23/12
$C0_5$	528	11X48	48/24 = 2/1

The Resolved Just Scale (RJS) is an 11-based refinement of traditional Just Intonation. It resolves intervallic gaps—such as the 33 Hz spacing—by subdividing them into 11 and 22 Hz steps, revealing a more continuous and structured harmonic field within the octave.

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Comparison of JI and RJS intervals					
JI Note	**JI Freq**	**Interval**	**RJS Note**	**RJS Freq**	**Interval**
$C0_4$	264		$C0_4$	264	
$C\sharp_{04}$	286	22	$C\sharp0_4$	286	22
$D0_4$	297	11	$D0_4$	297	11
$D1\sharp_4$	319	22	$D\sharp0_4$	319	22
$E0_4$	330	11	$E0_4$	330	11
$F0_4$	352	22	$F0_4$	352	22
$F\sharp0_4$	374	22	$F\sharp0_4$	374	22
$G0_4$	396	22	$G0_4$	396	22
$G\sharp0_4$	418	22	$G\sharp0_4$	418	22
$A0_4$	440	22	$A0_4$	440	22
$A\sharp0_4$	462	22	$A\sharp0_4$	462	22
			$B2_4$	**473**	**11**
$B0_4$	**495**	**33**	**$B0_4$**	**495**	**22**
			$B3_4$	**506**	**11**
$C0_5$	**528**	**33**	**$C0_5$**	**528**	**22**

Key Insight:

33 Hz intervals in Just Intonation resolve into 11 + 22 Hz in RJS.

Additional Notes/ Frequencies In the RJS System

In the Scale of 11, there are 5 frequencies between $A\sharp0_4$ and $C0_5$. They are 473, 484, 495, 506 and 517). The Just Intonation Scale using 1 frequency, 495. In the RJS system we add 2 additional notes when we covert the interval difference of 33 to 11 and 22, these being 473 and 506. Some consideration should be given to adding additional notes to the 12-note octave, since the RJS scale contains 14 unique notes and the Scale of 11 contains 24. But what name should be given to these new notes? Thoughts also needs to addressed to the Solfeggio notation. At this point I will keep the different variations of B ($B0_4$-$B4_4$)

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Summary

The Scale of 11 provides a generative framework from which tones emerge through sequential multiples of 11. The Resolved Just Scale (RJS) is an 11-based refinement of traditional Just Intonation. It resolves intervallic gaps—such as the 33 Hz spacing—by subdividing them into 11 and 22 Hz steps, revealing a more continuous and structured harmonic field within the octave.

Together, these systems distinguish between:

- **Discovery (Scale of 11)**
- **Application (RJS)**

Take aways:

What the Scale of 11 (S11) and the Resolved Just Intonation system (RJS) allows is:

- More precise harmonic relationships
- Expanded tonal resolution
- New compositional / therapeutic possibilities

The Resolved Just Scale reveals previously hidden tonal positions within the octave, expanding harmonic resolution beyond traditional systems. This creates new possibilities for musical structure, resonance, and applied sound work.

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